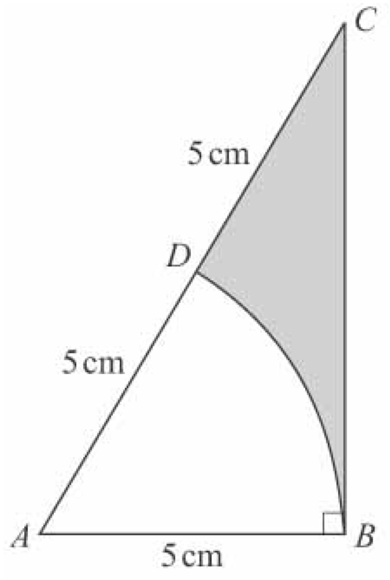
Chapter review 7

1



a In the right-angled triangle ABC :

$\cos \angle BAC = \frac{BA}{AC} = \frac{5}{10} = \frac{1}{2}$

so $\angle BAC = \frac{\pi}{3}$

b Area of triangle ABC

$= \frac{1}{2} \times AB \times AC \times \sin \angle BAC$

$= \frac{1}{2} \times 5 \times 10 \times \sin \frac{\pi}{3} = 21.650\ldots \text{cm}^2$

Area of sector DAB

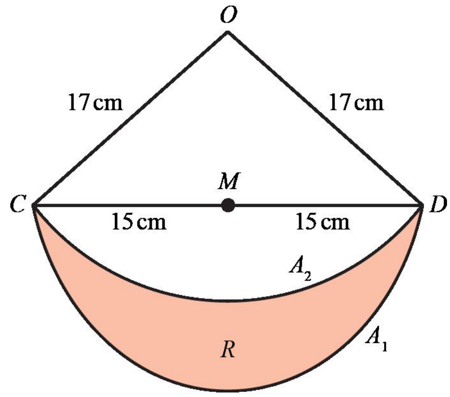
$= \frac{1}{2} \times 5^2 \times \frac{\pi}{3} = 13.089\ldots \text{cm}^2$

Area of shaded region

$=$ area of $\triangle\ ABC -$ area of sector DAB

$= 21.650\ldots - 13.089\ldots = 8.56 \text{ cm}^2$ (3 s.f.)

2



a Using Pythagoras' theorem to find OM :

$OM^2 = 17^2 - 15^2 = 64 \Rightarrow OM = 8 \text{cm}$

Area of $\triangle OCD = \frac{1}{2} \times CD \times OM$

$= \frac{1}{2} \times 30 \times 8 = 120 \text{ cm}^2$

b Area of shaded region R

$=$ area of semicircle CDA_1

$-$ area of segment CDA_2

Area of semicircle CDA_1

$= \frac{1}{2} \times \pi \times 15^2 = 353.429\ldots \text{cm}^2$

Area of segment CDA_2

$=$ area of sector OCD

$-$ area of triangle OCD

$= \frac{1}{2} \times 17^2 \times \angle COD - 120$

In right-angled triangle COM:

$\sin \angle COM = \frac{CM}{OC} = \frac{15}{17}$

so $\angle COM = 1.0808\ldots$

hence $\angle COD = 2.1616\ldots$

So area of segment CDA_2

$= \frac{1}{2} \times 17^2 \times 2.1616\ldots - 120$

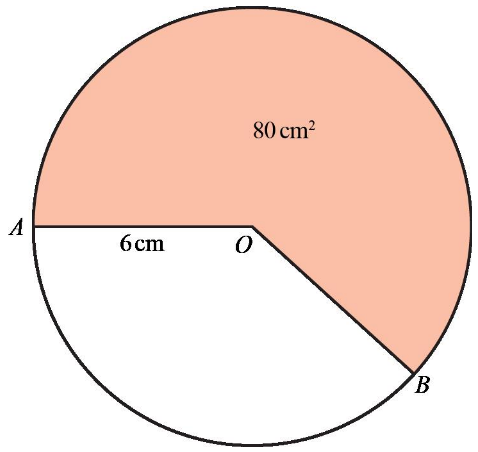
$= 192.362\ldots \text{cm}^2$

So area of shaded region R

$= 353.429\ldots - 192.362\ldots$

$= 161.07 \text{ cm}^2$ (2 d.p.)

3



a Reflex angle $AOB = (2\pi - \theta)$ rad
Area of shaded sector

$= \frac{1}{2} \times 6^2 \times (2\pi - \theta)$

$= (36\pi - 18\theta)\text{ cm}^2$

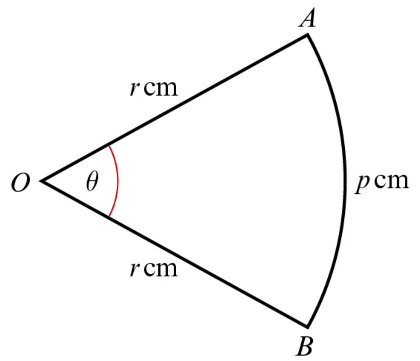
So $80 = 36\pi - 18\theta$

$\Rightarrow 18\theta = 36\pi - 80$

$\Rightarrow \theta = \frac{36\pi - 80}{18} = 1.839$ (3 d.p.)

b Length of minor arc AB
$= 6\theta = 6 \times 1.8387... = 11.03\text{ cm}$ (2 d.p.)

4



a Using $l = r\theta$:

$p = r\theta \Rightarrow \theta = \frac{p}{r}$

b Area of sector
$= \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times \frac{p}{r} = \frac{1}{2}pr\text{ cm}^2$

c $4.65 \leqslant r < 4.75$, $5.25 \leqslant p < 5.35$
Least possible value for area of sector

$= \frac{1}{2} \times 5.25 \times 4.65 = 12.207\text{ cm}^2$ (3 d.p.)

(Note: Least possible value is 12.20625, so 12.207 should be given, not 12.206)

d Maximum possible value of θ

$= \frac{\max p}{\min r} = \frac{5.35}{4.65} = 1.1505...$

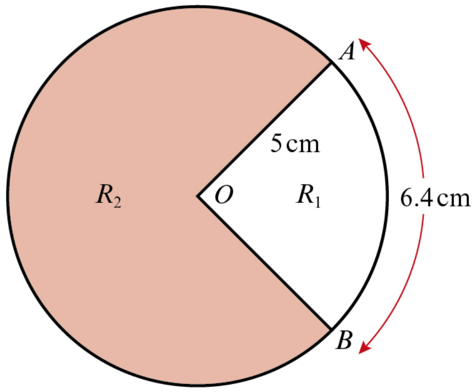
So give 1.150 (3 d.p.)

Minimum possible value of θ

$= \frac{\min p}{\max r} = \frac{5.25}{4.75} = 1.1052...$

So give 1.106 (3 d.p.)

5



a Using $l = r\theta$:

$6.4 = 5\theta \Rightarrow \theta = \frac{6.4}{5} = 1.28\text{ rad}$

b Using area of sector $= \frac{1}{2}r^2\theta$:

$R_1 = \frac{1}{2} \times 5^2 \times 1.28 = 16$

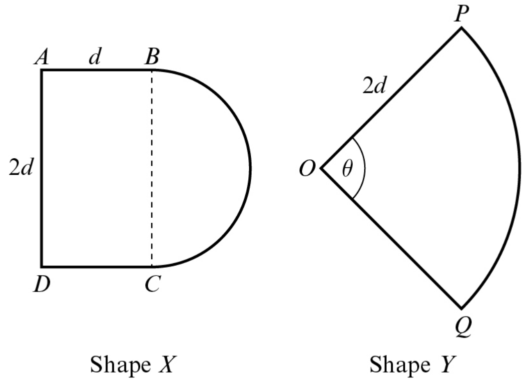
c R_2 = area of circle $- R_1$

$= \pi \times 5^2 - 16 = 62.5398...$

So $\frac{R_1}{R_2} = \frac{16}{62.5398...} = \frac{1}{3.908...} = \frac{1}{p}$

$\Rightarrow p = 3.91$ (3 s.f.)

6



a Area of shape X
$= \text{area of rectangle} + \text{area of semicircle}$
$= (2d^2 + \frac{1}{2}\pi d^2)\text{ cm}^2$

Area of shape $Y = \frac{1}{2}(2d)^2\theta = 2d^2\theta \text{ cm}^2$

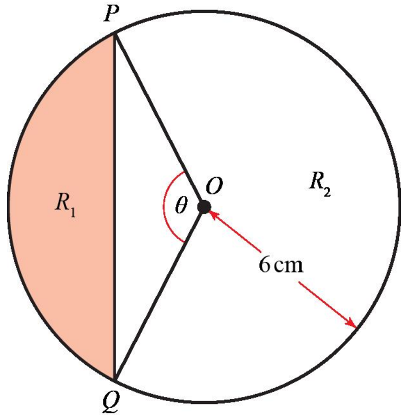
Since $X = Y$:
$2d^2 + \frac{1}{2}\pi d^2 = 2d^2\theta$
Divide by $2d^2$:
$1 + \frac{\pi}{4} = \theta$

b Perimeter of shape X
$= (d + 2d + d + \pi d)\text{ cm}$ with $d = 3$
$= (3\pi + 12)\text{ cm}$

c Perimeter of shape Y
$= (2d + 2d + 2d\theta)\text{ cm}$
with $d = 3$ and $\theta = 1 + \frac{\pi}{4}$
$= 12 + 6\left(1 + \frac{\pi}{4}\right)$
$= \left(18 + \frac{3\pi}{2}\right)\text{cm}$

d Difference
$= \left(18 + \frac{3\pi}{2}\right) - (3\pi + 12)$
$= 6 - \frac{3\pi}{2}$
$= 1.287\ldots\text{ cm}$
$= 12.9\text{ mm (3 s.f.)}$

7

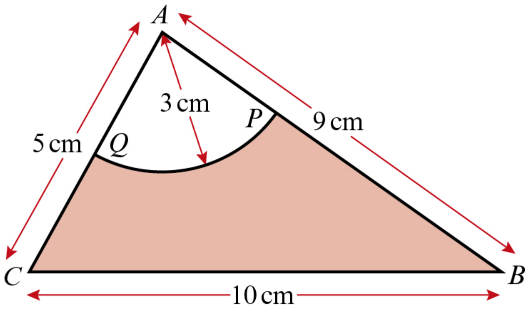


a Area of segment R_1
= area of sector OPQ
– area of triangle OPQ
$\Rightarrow A_1 = \frac{1}{2} \times 6^2 \times \theta - \frac{1}{2} \times 6^2 \times \sin\theta$
$\Rightarrow A_1 = 18(\theta - \sin\theta)$

b $A_2 = \text{area of circle} - A_1$
$= \pi \times 6^2 - 18(\theta - \sin\theta)$
$= 36\pi - 18(\theta - \sin\theta)$

Since $A_2 = 3A_1$:
$36\pi - 18(\theta - \sin\theta) = 3 \times 18(\theta - \sin\theta)$
$36\pi - 18(\theta - \sin\theta) = 54(\theta - \sin\theta)$
$36\pi = 72(\theta - \sin\theta)$
$\frac{\pi}{2} = \theta - \sin\theta$
$\sin\theta = \theta - \frac{\pi}{2}$

8



a Using the cosine rule in $\triangle ABC$:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\Rightarrow \cos \angle BAC = \frac{5^2 + 9^2 - 10^2}{2 \times 5 \times 9} = 0.0\dot{6}$$

$$\Rightarrow \angle BAC = 1.50408...$$

$$= 1.504 \text{ rad (3 d.p.)}$$

b i Using the sector area formula:

$$\text{area of sector} = \frac{1}{2}r^2\theta$$

$\Rightarrow$ area of sector APQ

$$= \frac{1}{2} \times 3^2 \times 1.504 = 6.77 \text{ cm}^2 \quad \text{(3 s.f.)}$$

ii Area of shaded region $BPQC$

= area of $\triangle ABC$ − area of sector APQ

$$= \frac{1}{2} \times 5 \times 9 \times \sin 1.504 - \frac{1}{2} \times 3^2 \times 1.504$$

$$= 15.681...$$

$$= 15.7 \text{ cm}^2 \quad \text{(3 s.f.)}$$

iii Perimeter of shaded region $BPQC$

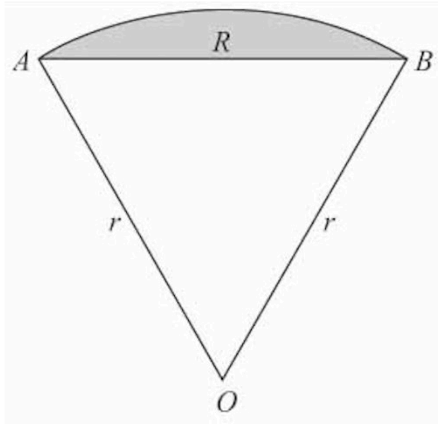
$= QC + CB + BP +$ arc length PQ

$$= 2 + 10 + 6 + (3 \times 1.504)$$

$$= 22.51...$$

$$= 22.5 \text{ cm} \quad \text{(3 s.f.)}$$

9



a Area of sector $= \frac{1}{2}r^2\theta = \frac{1}{2}r^2 \times 1.5 \text{ cm}^2$

So $\frac{3}{4}r^2 = 15$

$$\Rightarrow r^2 = \frac{60}{3} = 20$$

$$\Rightarrow r = \sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$$

b Arc length $AB = r(1.5) = 3\sqrt{5}$ cm

Perimeter of sector OAB

$= AO + OB +$ arc length AB

$$= 2\sqrt{5} + 2\sqrt{5} + 3\sqrt{5}$$

$$= 7\sqrt{5}$$

$$= 15.7 \text{ cm} \quad \text{(3 s.f.)}$$

c Area of segment R

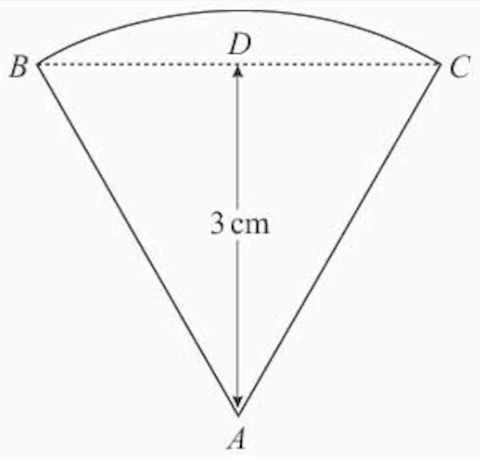
= area of sector − area of $\triangle AOB$

$$= 15 - \frac{1}{2}r^2 \sin 1.5$$

$$= 15 - 10 \sin 1.5$$

$$= 5.025 \text{ cm}^2 \text{ (3 d.p.)}$$

10



a Using the right-angled $\triangle ABD$, with

$\angle ABD = \frac{\pi}{3}$:

$\sin\frac{\pi}{3} = \frac{3}{AB}$

$\Rightarrow AB = \frac{3}{\sin\frac{\pi}{3}} = \frac{3}{\frac{\sqrt{3}}{2}}$

$= 3 \times \frac{2}{\sqrt{3}} = 2\sqrt{3}$ cm

b Area of badge = area of sector

$= \frac{1}{2} \times (2\sqrt{3})^2 \theta$ where $\theta = \frac{\pi}{3}$

$= \frac{1}{2} \times 4 \times 3 \times \frac{\pi}{3}$

$= 2\pi \text{ cm}^2$

c Perimeter of badge

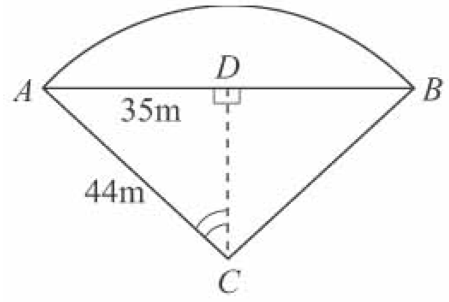
$= AB + AC +$ arc length BC

$= 2\sqrt{3} + 2\sqrt{3} + 2\sqrt{3} \times \frac{\pi}{3}$

$= 2\sqrt{3}\left(2 + \frac{\pi}{3}\right)$

$= \frac{2\sqrt{3}}{3}(\pi + 6)$ cm

11



a Using the right-angled $\triangle ADC$:

$\sin \angle ACD = \frac{35}{44}$

So $\angle ACD = \sin^{-1}\left(\frac{35}{44}\right)$

and $\angle ACB = 2\sin^{-1}\left(\frac{35}{44}\right)$

$\Rightarrow \angle ACB = 1.8395...$

$= 1.84$ rad (2 d.p.)

b i Length of railway track

= length of arc AB

$= 44 \times 1.8395...$

$= 80.9$ m (3 s.f.)

ii Shortest distance from C to AB is DC. Using Pythagoras' theorem:

$DC^2 = 44^2 - 35^2$

$DC = \sqrt{44^2 - 35^2} = 26.7$m (3 s.f.)

iii Area of region

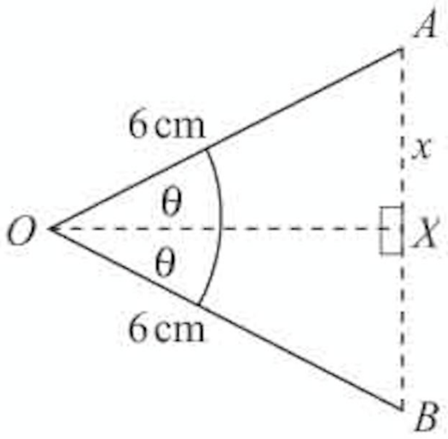
= area of segment

=area of sector ABC − area of $\triangle ABC$

$= \frac{1}{2} \times 44^2 \times 1.8395... - \frac{1}{2} \times 70 \times DC$

$= 847\text{m}^2$ (3 s.f.)

12



a In right-angled $\triangle OAX$ (see diagram):

$\frac{x}{6} = \sin\theta$

$\Rightarrow x = 6\sin\theta$

So $AB = 2x = 12\sin\theta$ ($AB = DC$)

Perimeter of the cross-section

$= \text{arc length } AB + AD + DC + BC$

$= 6 \times 2\theta + 4 + 12\sin\theta + 4$

$= (8 + 12\theta + 12\sin\theta)$ cm

So $2(7 + \pi) = 8 + 12\theta + 12\sin\theta$

$\Rightarrow 14 + 2\pi = 8 + 12\theta + 12\sin\theta$

$\Rightarrow 12\theta + 12\sin\theta - 6 = 2\pi$

Divide by 6:

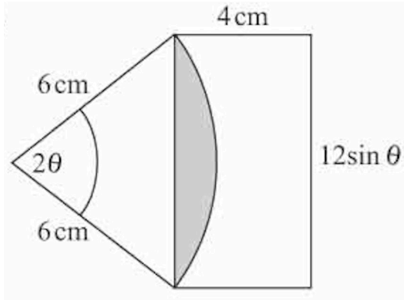
$2\theta + 2\sin\theta - 1 = \frac{\pi}{3}$

b When $\theta = \frac{\pi}{6}$,

$2\theta + 2\sin\theta - 1 = \frac{\pi}{3} + \left(2 \times \frac{1}{2}\right) - 1$

$= \frac{\pi}{3}$

c



Area of the cross-section
$=$ area of rectangle $ABCD$
$-$ area of shaded segment

Area of rectangle $= 4 \times 12 \times \sin\frac{\pi}{6}$

$= 24\text{ cm}^2$

Area of shaded segment
$=$ area of sector $-$ area of triangle

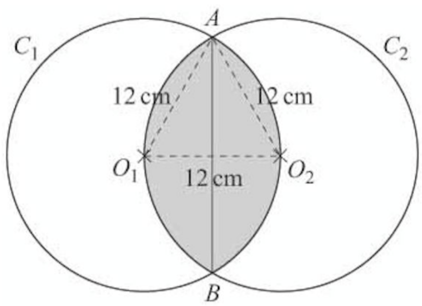
$= \frac{1}{2} \times 6^2 \times \frac{\pi}{3} - \frac{1}{2} \times 6^2 \times \sin\frac{\pi}{3}$

$= 3.261\ldots\text{cm}^2$

So area of cross-section

$= 20.7\text{ cm}^2$ (3 s.f.)

13



a $O_1A = O_2A = 12$, as they are radii of their respective circles.
$O_1O_2 = 12$, as O_2 is on the circumference of C_1 and hence is a radius (and vice versa).
Therefore $\triangle AO_1O_2$ is equilateral

So $\angle AO_1O_2 = \frac{\pi}{3}$

and $\angle AO_1B = 2 \times \angle AO_1O_2 = \frac{2\pi}{3}$

13 b Consider arc AO_2B of circle C_1.
Using arc length $= r\theta$:

arc length $AO_2B = 12 \times \frac{2\pi}{3} = 8\pi$ cm

Perimeter of R
$=$ arc length AO_2B + arc length AO_1B
$= 2 \times 8\pi = 16\pi$ cm

c Consider the segment AO_2B in circle C_1.
Area of segment AO_2B
$=$ area of sector O_1AB – area of $\triangle O_1AB$

$= \frac{1}{2} \times 12^2 \times \frac{2\pi}{3} - \frac{1}{2} \times 12^2 \times \sin\frac{2\pi}{3}$

$= 88.442...\text{cm}^2$
Area of region R
$=$ area of segment AO_2B
  $+$ area of segment AO_1B
$= 2 \times 88.442...$
$= 177\text{ cm}^2$ (3 s.f.)

14 a The student has used an angle measured in degrees – it needs to be measured in radians to use that formula.

b $50° = \frac{50}{180} \times \pi$ rad

$\frac{1}{2}r^2\theta = \frac{1}{2} \times 3^2 \times \frac{5}{18}\pi$

$= \frac{5}{4}\pi\text{ cm}^2$

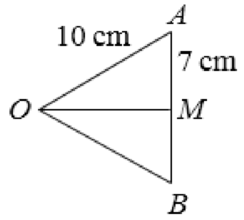
When is small:
LHS $\approx 4\theta$

and RHS $\approx 37 - 2\left(1 - \frac{(2\theta)^2}{2}\right)$

so $4\theta = 37 - 2 + 4\theta^2$
$4\theta^2 - 4\theta + 35 = 0$
$b^2 - 4ac < 0$
So there are no solutions.

Challenge

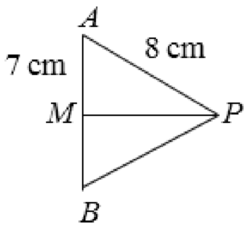
a Let the centre of the larger circle be O and the midpoint of AB be M.
The right-angled triangle OAM has sides $OA = 10$ cm and $AM = 7$ cm



To find the size of the angle AOM,

$\sin AOM = \frac{7}{10}$

$AOM = 0.7753\ldots$ radians
Since $AOB = 2AOM$
$AOB = 1.5507\ldots = 1.551$ radians (3 d.p.)
Similarly, let the centre of the smaller circle be P.
The right-angled triangle PAM has sides $PA = 8$ cm and $PM = 7$ cm



To find the size of the angle APM,

$\sin APM = \frac{7}{8}$

$APM = 1.0654\ldots$ radians
Since $APB = 2APM$
$APB = 2.1308\ldots = 2.131$ radians (3 d.p.)

b Area of sector $APB = \frac{1}{2}r^2\angle APB$

$= \frac{1}{2}\times 8^2\times 2.1308\ldots$

$= 68.187\ldots \text{ (cm}^2\text{)}$

Area of triangle $APB = \frac{1}{2}ab\sin\angle APB$

$= \frac{1}{2}\times 8\times 8\times \sin 2.1308\ldots$

$= 27.110\ldots \text{ (cm}^2\text{)}$

Area of triangle $AOB = \frac{1}{2}ab\sin\angle AOB$

$= \frac{1}{2}\times 10\times 10\times \sin 1.5507\ldots$

$= 49.989 \text{ (cm}^2\text{)}$

Finally, the shaded area **R** is found by adding the area of triangle APB and the area of triangle AOB and subtracting the area of sector APB:
Area **R** $= 27.110\ldots + 49.989\ldots - 68.187\ldots$
$= 8.91 \text{ cm}^2$ (3 s.f.)